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Source: *The Review of Economic Studies*, Vol. 46, No. 4 (Oct., 1979), pp. 571-585

Published by: Oxford University Press

Stable URL: <http://www.jstor.org/stable/2297028>

Accessed: 28-08-2017 22:08 UTC

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# Equity, Efficiency and Increasing Returns

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## 1. INTRODUCTION

We shall be concerned in this paper with certain properties of equilibria in which firms with increasing returns to scale in production sell their outputs at prices equal to marginal costs. We are in other words concerned with equilibria involving increasing returns and at which the standard first-order or marginal conditions for optimality are met. Figures (1a) to (1c) illustrate such equilibria in a two-good economy. It is not our concern to prove the existence of such equilibria: this we have done elsewhere (Brown and Heal (1976)) and existence is here assumed. Indeed, the paper will focus on examples sufficiently simple that no formal proof of existence is required. Another very complex issue that will not be our concern, is the possibility of supporting and implementing an equilibrium of this sort. There is of course a problem here, because it may require some firms to operate at a loss, which will then have to be covered by non-distortionary levies on other agents if the first-order conditions are to be preserved. Again, this is a matter that we have discussed elsewhere (Brown and Heal (1979)), demonstrating that such an equilibrium can be sustained without the need for conventional lump-sum taxes. Because it is not at present our main concern, we deal with the problem here in a slightly cavalier way, and assume all shareholdings to involve unlimited liability. The owners of a firm therefore share both in its profits and its losses, and losses are covered from shareholders' incomes. In the examples considered below there is a single firm, owned entirely by one of two consumers, and it will become clear that at an efficient equilibrium it would not be in the owner's interest to close it down, even though it makes a loss. This is just a particular example of a more general proposition discussed in Brown and Heal (1979): the point is that the income saving from closure would be outweighed by the loss of consumer surplus otherwise provided by having the goods available at marginal cost.

The focus of this paper, then, is not on the existence or decentralization of an equilibrium satisfying the first-order conditions for optimality, but rather on its optimality. It is clear from Figure 1 that such an equilibrium need not be optimal, for the obvious reason that with non-convex production possibility sets, the first-order conditions are not sufficient to ensure optimality. We shall not in fact be concerned with an analysis of the sufficiency of these conditions (for this, see Brown and Heal (1979)), our concern is rather with the question: is it the case that there is always at least one equilibrium which satisfies the first-order conditions and is Pareto optimal?

The earlier diagrams, and one's intuition, suggest an affirmative answer. In fact Guesnerie (1975), in a seminal paper on general equilibrium and non-convex production sets, has already shown the answer to be negative: there may be economies with many

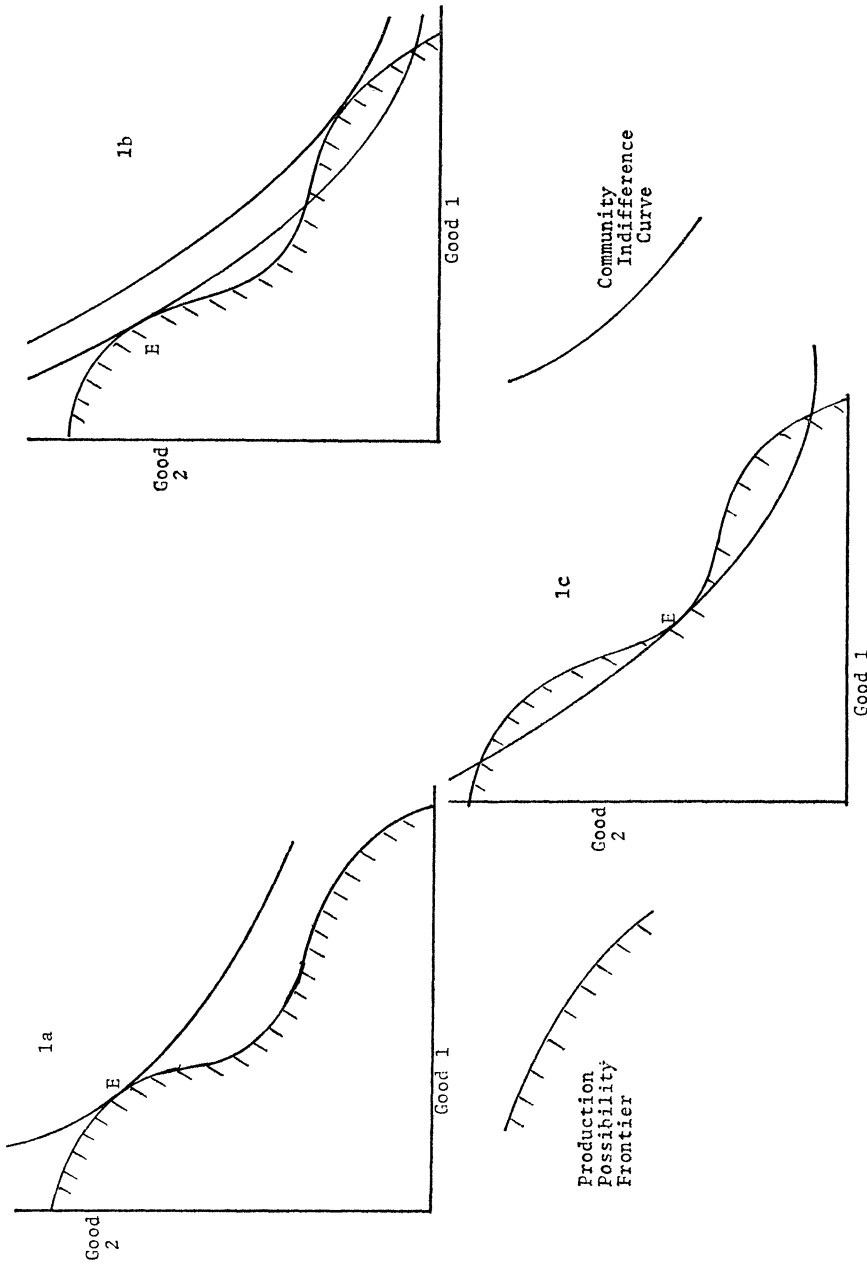


FIGURE 1

equilibria, all of which satisfy the first-order conditions and yet are inefficient. Our purpose here is to analyse an example which makes it possible to illustrate this point more fully and simply than has been done hitherto, and also to set out some of the very significant implications that this finding has for the welfare economics of economies with increasing returns—which include, of course, most economies for which welfare economics is ever likely to be practised. Finally, we shall mention certain conditions sufficient to ensure that this problem does not arise. These conditions will imply the existence of at least one efficient equilibrium, and thus in a certain sense guarantee that the system is well-behaved. Perhaps we should emphasize that an equilibrium of the type we discuss can be inefficient only if firms are not maximizing profits at the “marginal cost” prices. Of course, this will often be the case with non-convexities. If firms are at positions of maximum profits, one has a standard competitive equilibrium for which the usual results are true.

In particular, we shall show that the answer to the question “Is there always an efficient equilibrium?” depends upon the distribution of initial endowments of both goods and services and of claims upon the profits (or liabilities to the losses) of firms. For some such distributions, the answer is affirmative, and for others, negative. That is, for a given set of firms and consumers, and a given total of initial physical endowments, some distributions of that total and of claims upon firms will ensure the existence of at least one Pareto optimal equilibrium, whereas other distributions will ensure that no equilibrium is optimal. We thus have to recognize that some distributions are efficient (in the sense of permitting the attainment of optimality), and others are not. This is in very sharp distinction to the standard competitive case, where one judges between alternative distributions purely on the grounds of equity, confident that the efficiency or otherwise of an outcome resulting from a given endowment distribution is purely a function of the allocative system used to achieve it. It is thus a standard proposition in the Arrow–Debreu model that, given any distribution of endowments satisfying certain technical conditions, there exists an associated competitive equilibrium which is efficient. Equity and efficiency are therefore independent dimensions, and much of our accepted welfare economics and cost-benefit analysis rests, explicitly or implicitly, on this fact. The examples we present below demonstrate that, once one admits increasing returns, the situation is fundamentally different. Because some are efficient and others inefficient, one can no longer judge between alternative distributions purely in terms of equity. It is necessary to consider both the equity and the efficiency dimensions simultaneously.

At the risk of seeming repetitive, we put this point in another way. In a world where firms display non-increasing returns in production, a suggested pattern of endowments of goods and services and of claims on firms, provided it permits the existence of an equilibrium, can be rejected only on the grounds of a judgement that it is inequitable. With increasing returns, however, one has also the option of rejecting some proposed endowment patterns on the grounds that they are inefficient. We shall go on to demonstrate an even more surprising point, namely, that with increasing returns in production, it may be possible to remove some endowment from one person, give it to another, and make both better off. In other words, an unrequited transfer of property from one agent to another—an expropriation—may make both better off. Though paradoxical at first sight, this point is in fact easily understood once one realizes that there may be efficient and inefficient distributions of endowments. The kind of paradox just referred to arises when the initial distribution is inefficient, but the distribution after the unrequited transfer is efficient. In such a case, the transfer will move the economy from an equilibrium inside the utility possibility frontier, to one on the frontier, and it is possible that in the process all may gain.

A natural reaction to these paradoxical results, is to enquire whether they are general properties of all economies with increasing returns. While we are far from being in a position to give a complete answer on this point, in the later sections of the paper we do present a condition weaker than convexity which is sufficient to ensure that there is always an efficient equilibrium. This exploits the intuitively obvious fact that in a one-person

economy the problem cannot occur, to show that if individuals' preferences are in a certain sense similar (i.e. can be aggregated), then there is always an efficient outcome. We also mention certain rather complex topological conditions on the economy sufficient to prevent this paradox.

These observations about the interrelationship between equity and efficiency, and the possibility of unrequited Pareto-improving transfers, are somewhat surprising, and not readily linked to the earlier literature in this field. It may, however, be worth noting that there is a line of argument that has been advanced in development economics and mentioned to us by people in that area, which is reminiscent of our conclusions. This is an argument to the effect that in an under-developed country with a highly unequal distribution of income, a redistribution away from the very rich may in the long-run make all better off, because the acquisition of purchasing power by the middle and lower income groups may lead to the development of a mass market and a substantial increase in industrial profits. It seems that such an argument has not been formalized, but that increasing returns in production would be an essential ingredient of any attempt to do so.

The remainder of the paper is divided into five sections. In Section 2 we outline the type of model and of argument to be used. In Section 3 we give a geometric, and in Section 4 a mathematical, treatment of the model. Both are included because the geometric analysis probably makes the intuitive basis of the work much clearer, but a formal mathematical treatment seems needed to confirm the results suggested by the diagrams. In Section 5 we develop the alternative conditions sufficient to ensure the existence of at least one efficient equilibrium, and in Section 6 we survey the implications of the work, and the major unanswered questions.

## 2. AN OUTLINE

We turn now at some length to the first issue raised above, and discuss how to demonstrate that there are economies for which there is *no* Pareto efficient equilibrium. We shall also show that this inefficiency of the equilibrium is very sensitive to the distribution of endowments. The basic intuition may be given as follows. Figure 2 shows an economy for which the feasible set  $Y$  is non-convex.

Clearly the equilibria here must be at A or B. (There may be an equilibrium at G, but as this is clearly inefficient we ignore it.) We suppose there to be two consumers (I and II) and draw the Scitovsky community indifference curve through A. This passes inside B, which is therefore Pareto-superior to A. It is therefore natural to suppose B to be Pareto-efficient. But in moving to B, relative prices change, as therefore do the distributions of income and wealth. The community indifference curves corresponding to this new distribution will in general intersect those corresponding to the previous distribution, and indeed the curve drawn through B has this property. It furthermore passes inside A, which is now Pareto-superior to B. Hence neither point is Pareto-efficient, and there is therefore no efficient equilibrium.

We shall show below how to construct an economy for which the above phenomenon occurs. Our approach will be primarily geometrical, and it will be useful if we outline in advance the way the argument will be developed. We consider a two-person, two-good model with a production set  $Y$  as represented in Figure 2. The first step will be to construct the utility possibility frontier for this system. To do this, consider an equilibrium at point A. The possible efficient distributions of the net output represented by A amongst the two consumers are shown by the contract curve of an Edgeworth box OCAD in Figure 3. Given a numerical representation of the two preferences, this contract curve can be transposed to a curve in utility space. The curve MN in Figure 4 thus corresponds to the contract curve OA in Figure 3. It is clear that the equivalent curve corresponding to production at any other point in the rectangle OCAD will not lie anywhere above MN, as all other points in the rectangle correspond to smaller commodity bundles. We can now

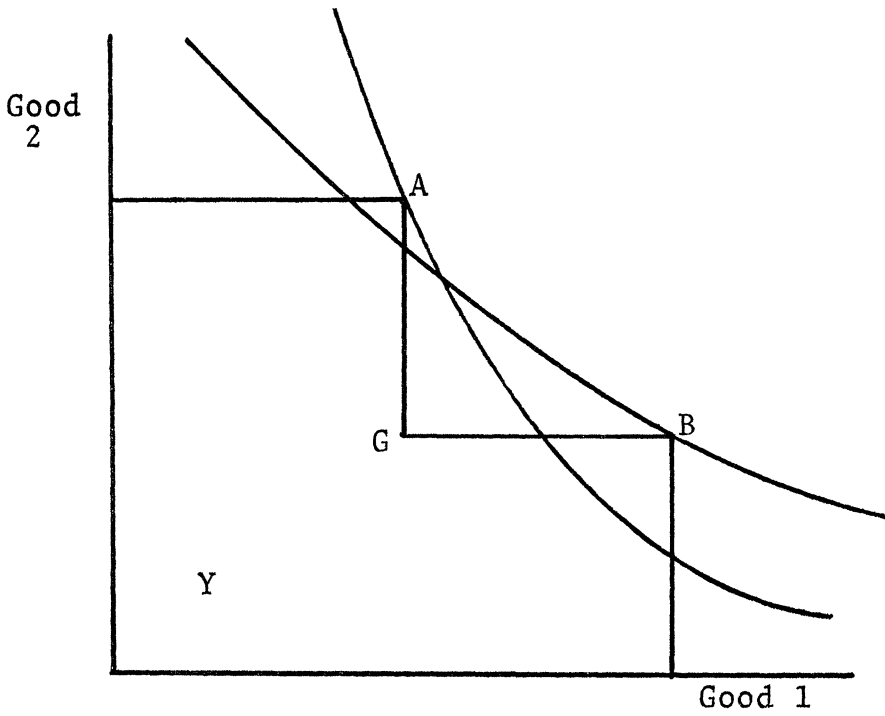


FIGURE 2

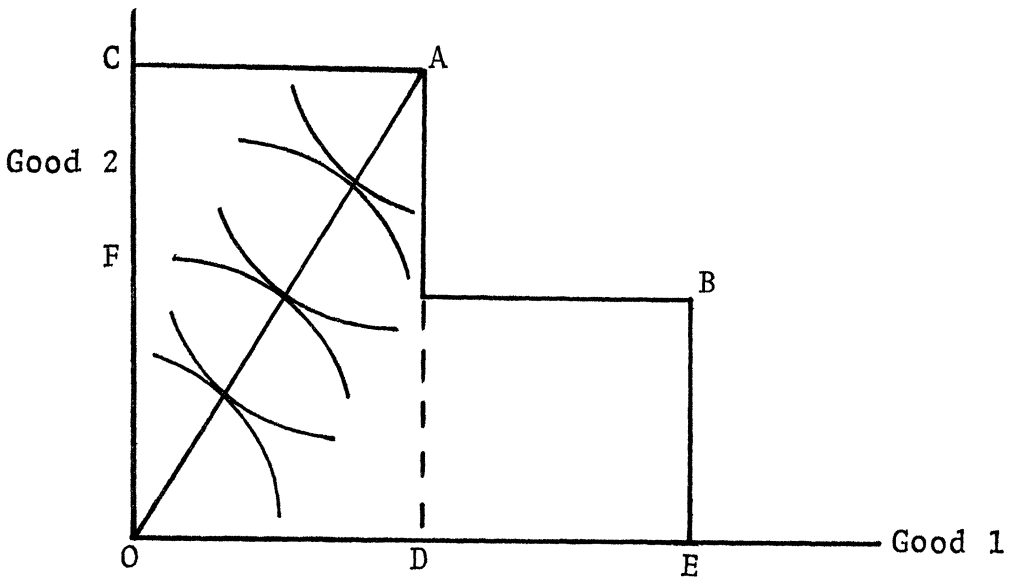


FIGURE 3

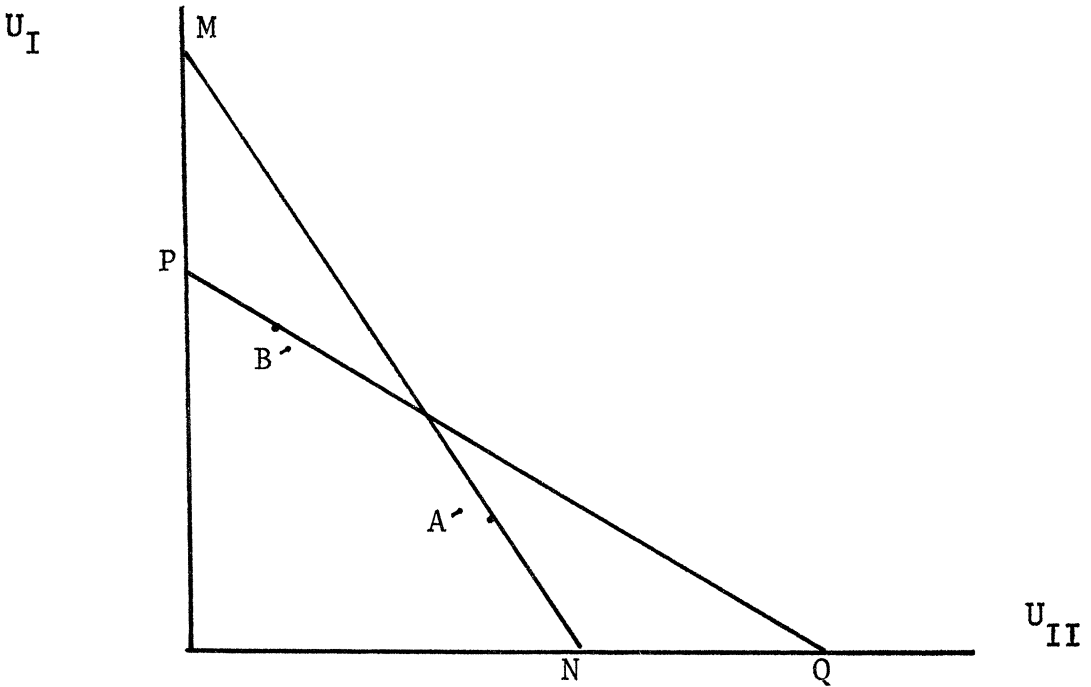


FIGURE 4

repeat this exercise for production at point B, using OFBE as an Edgeworth box. The line PQ in Figure 4 corresponds to this new contract curve, and once again it is clear that this dominates the corresponding line for production at any other point in the rectangle OFBE. Hence the utility possibility frontier is the outer envelope of MN and PQ.

All that now remains to be shown is that the distribution of utilities that occurs in an equilibrium at A corresponds to an inefficient point such as A' on MN, and likewise that an equilibrium at B leads to B' on PQ. When we have done this, we shall also be able to show that different patterns of initial endowments give rise to equilibria on the utility possibility frontier, some of which are Pareto-superior to A' or B'. This will of course imply:

- (a) that certain endowment patterns are, in the context of the present equilibrium concept, less efficient than others; and
- (b) that an unrequited transfer of endowments from one consumer to another may in certain circumstances make it possible to make both better off.

### 3. A GEOMETRIC ANALYSIS

We shall consider an economy with two goods,  $x$  and  $y$ ,  $y$  an output produced from  $x$  according to

$$y = \begin{cases} 0, & \text{if } x < 7 \\ 7, & \text{if } x \geq 7. \end{cases} \quad \dots(1)$$

The two individuals, I and II, have utility functions given by

$$U_I = \begin{cases} (y + \frac{4}{3}x) \cdot \frac{3\sqrt{2}}{7} & \text{if } y \geq x \\ (y + 0.04x) \cdot \frac{2 \cdot 24(3\sqrt{2})}{7} & \text{if } y \leq x \end{cases} \quad \dots(2)$$

$$U_{II} = \begin{cases} (3y + 5x) \frac{\sqrt{10}}{18} & \text{if } y \geq \frac{1}{3}x \\ \frac{x\sqrt{10}}{3} & \text{if } y = \frac{1}{3}x \\ \sqrt{y10} & \text{if } y \leq \frac{1}{3}x. \end{cases} \dots(3)$$

In spite of their apparent complexity, these are really very simple functions. They give piece-wise linear indifference curves with slopes of, in the case of I,  $-4/3$  and  $-4/100$ , and in the case of II,  $-5/3$  and  $0$ . These join along the lines  $y = x$  and  $y = x/3$  respectively, and the particular numerical representations chosen ensure that the utility level of any indifference curve is given by the distance from the origin to the point where it crosses either  $y = x$  or  $y = x/3$  respectively.

Initial endowments are

$$W_I = (0, 5) \quad W_{II} = (15, 0)$$

so that the total endowment is  $(15, 5)$ . It remains to specify ownership of the firm: this is owned entirely by I, and all share-holdings carry unlimited liability. I therefore receives all profits and meets all losses.

If  $Y'$  is the economy's production possibility set, then Figure 5 shows

$$Y = (Y' + W) \cap R_{\geq 0}^2.$$

The two possible equilibria are evidently at B and D in Figure 5, and it is clear that B is a point at which no production occurs. There is therefore one equilibrium where the only activity is trade in the initial endowment, and a second equilibrium with production.

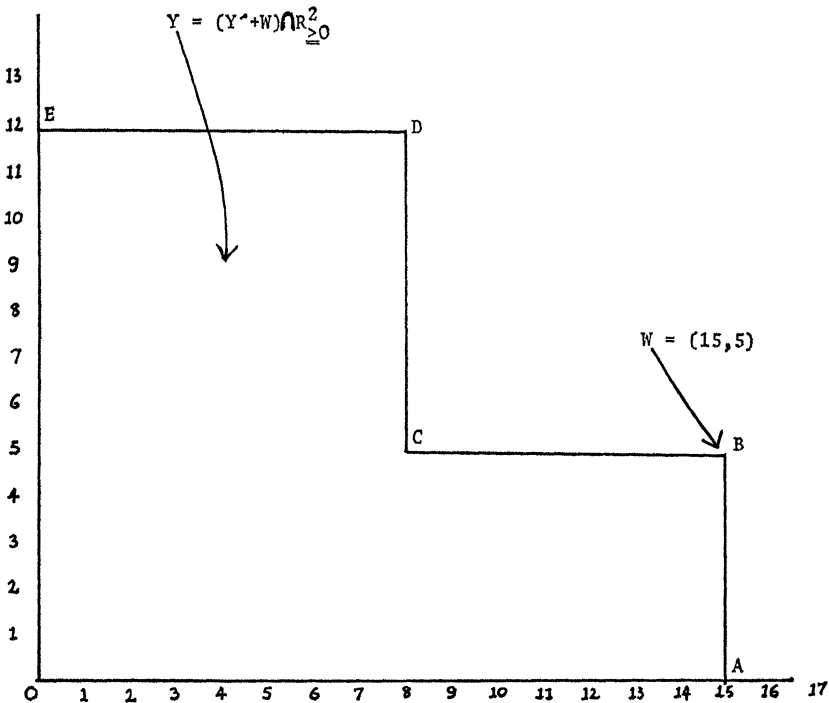


FIGURE 5



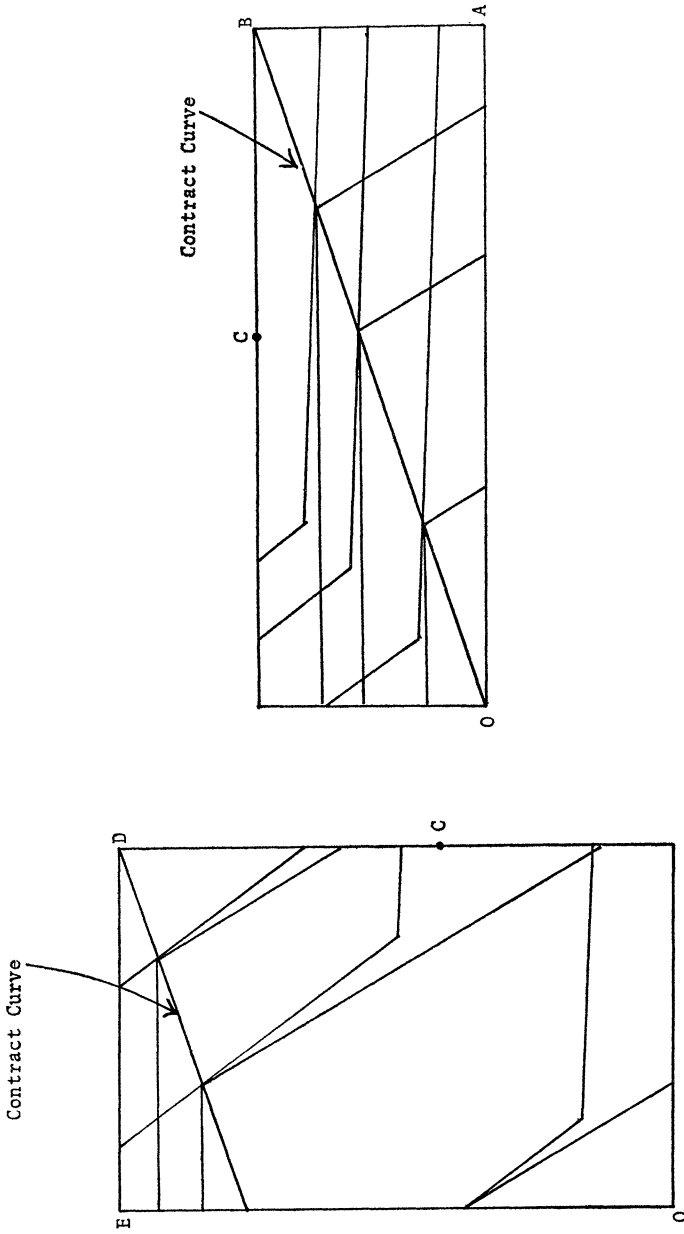


FIGURE 6  
Edgeworth boxes for production at D and at B.

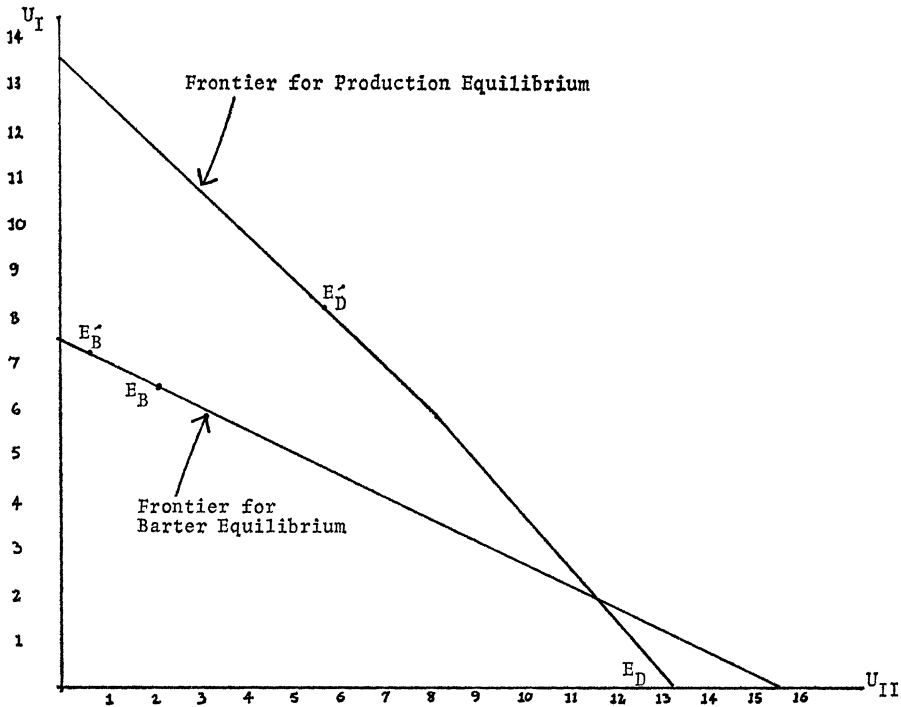


FIGURE 7  
Utility possibility frontier.

Figure 6 shows the Edgeworth boxes corresponding to production at points D and B, respectively: the piece-wise linear contract curves are indicated. In Figure 7 we see the utility possibility curves implied by these contract curves: the utility possibility frontier is, by our earlier arguments, the outer envelope of these.

In Figure 8, we see the two equilibrium distributions corresponding to production at D or at B: these are labelled  $E_D$  and  $E_B$ , respectively. The endowments of the two individuals are indicated by  $W_I$  and  $W_{II}$ , and as B involves no production and hence neither profit nor loss, I's budget line must pass through  $W_I$ . The contract curve is OB, and the equilibrium price ratio must equal the slope of I's indifference curves in the region  $x \geq y$ . Given this information and the location of one point on I's budget line, it is clear that the equilibrium is at  $E_B$ .

In the case of production at D, the contract curve consists of the segments OF and FD. Equilibrium occurs along OF, with prices given by the slope of II's indifference curves in this region. In this case production is occurring, and the firm will make a loss given, in terms of output, by the vertical distance  $B\pi$ . In computing I's budget line, we have to remember his responsibility for this loss: his budget line will therefore have the slope of II's indifference curves, with a vertical intercept  $B\pi$  below  $W_I$ . Such a line intersects the contract curve OFD at  $E_D$ , confirming that this is the equilibrium distribution. One can check the validity of this construction by projecting this budget line onwards to intersect the extension of DE, which it does to a point 15 units to the left of D. This is II's endowment in terms of  $x$ , so that the line is also II's budget line.

The utility levels corresponding to  $E_B$  and  $E_D$  are marked on the corresponding utility possibility curves in Figure 7. Clearly both configurations are Pareto inefficient, confirming that we have indeed constructed an economy for which all equilibria are inefficient. {Recall that utility values for I and II are given by distances along the lines  $y = x$  and OB, respectively.}

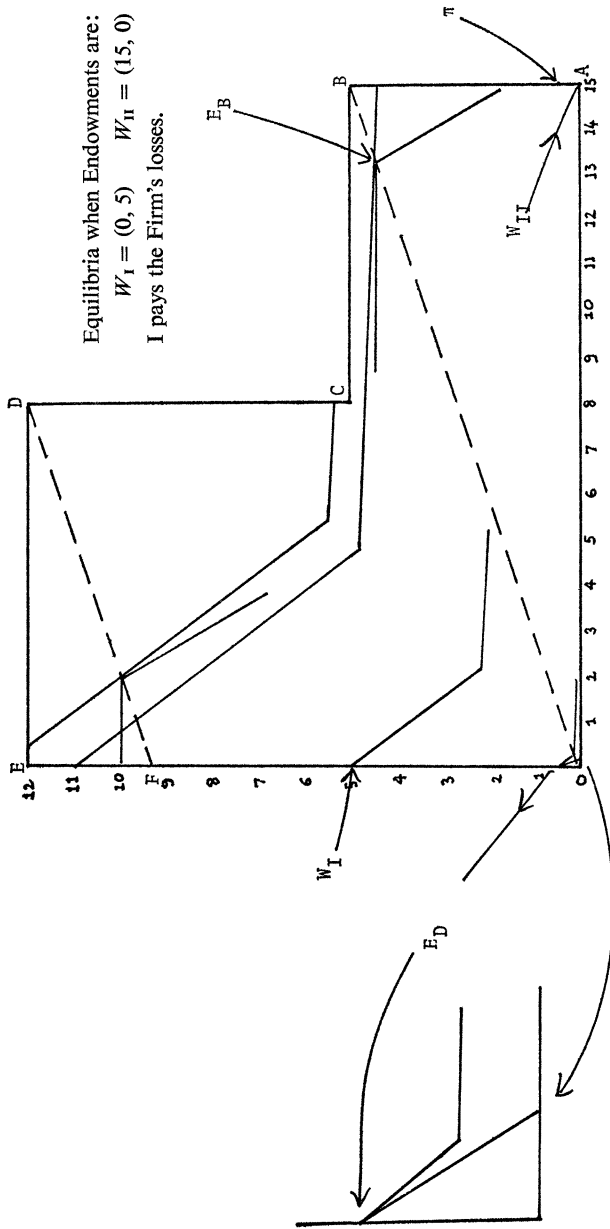


FIGURE 8

Equilibria when Endowments are:  
 $W_I = (0, 5)$      $W_{II} = (15, 0)$   
 I pays the Firm's losses.

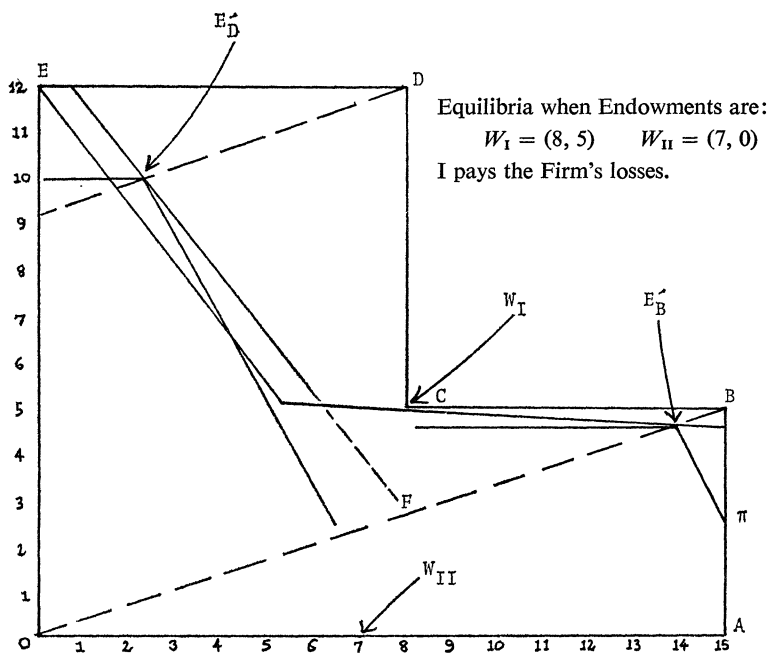


FIGURE 9

In Figure 9 we analyse the equilibria corresponding to a different distribution of endowments. These are now

$$W_I = (8, 5) \quad W_{II} = (7, 0)$$

so that an unrequited transfer of 8 units of  $x$  has been made from II to I. I continues to be the sole, unlimited liability, shareholder. These new endowments are shown in the figure, and the new equilibrium distributions,  $E'_B$  and  $E'_D$  are computed as before. The corresponding utility pairs are shown in Figure 7, where it is clear that  $E'_D$  is Pareto efficient, and indeed is superior to both  $E'_B$  and  $E_B$ .

It is worth emphasizing two conclusions that are apparent from Figure 7. The first is that at equilibrium  $E'_D$ , although I is responsible for meeting the losses of the firm, it would not be to his advantage to close it. This would result in a move to the equilibrium  $E'_B$ , at which both parties are worse off. The second point to note is that if the economy is initially at equilibrium  $E_B$ , a transfer of resources from II to I may move it to  $E'_D$ , making both I and II better off. We say may only because there is nothing in the model to indicate which of the various possible equilibria will be selected at any pattern of endowments.

Another point that we should perhaps note before moving on is that, although in this section we have demonstrated the possibility of removing the inefficiency by a change in physical endowments, there is nothing particularly special in this respect about the physical component of an individual's total endowment. We leave it as an exercise for the reader to verify that if the individuals' endowments of goods 1 and 2 were unaltered, but the ownership of the firm were transferred to individual II, then the production equilibrium would be efficient. It may therefore be possible to reach efficiency from an initially inefficient pattern of endowments by changing the ownership of financial assets—in effect by changing the liability for financing the firm's deficit. More generally, we might note that there are three components to the distribution of endowments in this model:

- (i) the fraction  $\alpha_i$  of good  $i$ ,  $i = 1, 2$ , owned by individual I,
- (ii) the fraction  $\alpha_3$  of the firm's profits or losses which accrue to individual I.

The set of all possible endowments can therefore be described by the unit cube in  $\mathbb{R}^3$ , and we have demonstrated that in this cube the point (0, 1, 1) is inefficient, whereas (8/15, 1, 1) and (0, 1, 0) are efficient. It has also been pointed out to us by Guesnerie that in our example all points of the form (x, x, x) are efficient. Such points correspond to situations where each person has a uniform share of every form of property, so that the distributional rule consists essentially of dividing the national income in fixed proportions. This is what Guesnerie (1975) refers to as an economy with a fixed structure of revenues. Unfortunately his own example (1975, Appendix) demonstrates that it is not in general true that all endowments of the form (x, x, x) are efficient. In his example, the point ( $\frac{1}{2}$ ,  $\frac{1}{2}$ ,  $\frac{1}{2}$ ) is inefficient.

4. MATHEMATICAL DERIVATION

We shall now verify the results suggested diagrammatically by computing the demand and supply functions for the economy considered there, and substituting into these the equilibrium prices suggested by the diagrams. This will enable us to verify that these are indeed equilibrium prices, and to compute precisely the associated consumption and utility levels. We shall also calculate the exact position of the utility possibility frontier.

*Demand Functions*

For individual I we have:

$$U_I = \begin{cases} \frac{3\sqrt{2}}{7} (y + \frac{4}{3}x) & \text{if } y \geq x \\ 2 \cdot 24 \frac{3\sqrt{2}}{7} (y + 0 \cdot 04x) & \text{if } y \leq x \end{cases} \dots(4)$$

and the demand functions are

$$-\frac{4}{3} < -\frac{Px}{Py} < -\frac{4}{100}, \quad x = y = \frac{Y_I}{Px + Py} \dots(5a)$$

$$\frac{Px}{Py} = \frac{4}{3}, \quad y \geq x \text{ and } 4x + 3y = Y_I \dots(5b)$$

$$\frac{Px}{Py} = \frac{4}{100}, \quad y \leq x \text{ and } 4x + 100y = Y_I \dots(5c)$$

$$-\frac{Px}{Py} < -\frac{4}{3}, \quad x = 0 \text{ and } y = \frac{Y_I}{Py} \dots(5d)$$

$$-\frac{Px}{Py} > -\frac{4}{100}, \quad y = 0 \text{ and } x = \frac{Y_I}{Px} \dots(5e)$$

Likewise, for II

$$U_{II} = \begin{cases} \frac{\sqrt{10}}{18} (3y + 5x) & \text{if } y \geq \frac{1}{3}x \\ \frac{\sqrt{10}}{3} x & \text{if } y = \frac{1}{3}x \\ \sqrt{10}y & \text{if } y \leq \frac{1}{3}x \end{cases} \dots(6)$$

and the demand functions are

$$0 > -\frac{Px}{Py} > -\frac{5}{3}, \quad y = \frac{Y_{II}}{3P_x + P_y} \quad \dots(7a)$$

$$x = 3y$$

$$\frac{Px}{Py} = \frac{5}{3}, \quad y \geq \frac{1}{3}x \text{ and } 5x + 3y = Y_{II} \quad \dots(7b)$$

$$-\frac{Px}{Py} < -\frac{5}{3}, \quad x = 0 \text{ and } y = \frac{Y_{II}}{P_y} \quad \dots(7c)$$

*Equilibria with  $W_I = (0, 5)$  and  $W_{II} = (15, 0)$*

We consider first the barter equilibrium  $E_B$  of Figure 8. The figure suggests as equilibrium prices  $P_x = 4$  and  $P_y = 100$ . In this case incomes are given by  $Y_I = 500$  and  $Y_{II} = 60$ . I's demand is indeterminate (by (5c)) and II's is given by (7a). Hence II's demands are, using obvious notation

$$D_{II}(x) = 1.607, \quad D_{II}(y) = 0.536.$$

Given the total endowments of 5 and 15 respectively, I's consumptions must be 13.393 and 4.464 for  $x$  and  $y$ . It is easily verified that these satisfy (5c), so that the prices are indeed market-clearing, given the supplies available at point B. Utility levels can then be calculated, giving as the overall configuration at  $E_B$ :

$$D_I(x) = 13.393, \quad D_I(y) = 4.464, \quad U_I = 6.787$$

$$D_{II}(x) = 1.607, \quad D_{II}(y) = 0.536, \quad U_{II} = 1.694.$$

These utility levels correspond to those shown in Figure 7. We can now turn to the production equilibrium  $E_D$  of Figure 8, for which the diagram suggests  $P_x = 5$  and  $P_y = 3$ . In this case  $Y_{II} = 75$ . The value of I's endowments is 15, but he also has to meet the operating losses associated with production by the firm at point D. These losses are 14, so I's net income is  $Y_I = 1$ .

I's demand is given by (5d), so that

$$D_I(x) = 0, \quad D_I(y) = \frac{1}{3}$$

II's demands are indeterminate by (7b). Total supplies of  $x$  and  $y$  are 8 and 12 respectively, giving

$$D_{II}(x) = 8, \quad D_{II}(y) = 11\frac{2}{3}$$

These satisfy (7b), so that we again have an equilibrium. The total configuration is:

$$D_I(x) = 0, \quad D_I(y) = \frac{1}{3}, \quad U_I = 0.202$$

$$D_{II}(x) = 8, \quad D_{II}(y) = 11\frac{2}{3}, \quad U_{II} = 13.176$$

and these utilities correspond to those shown at  $E_D$  in Figure 7. The remaining computations are similar, and are now left to the interested reader.

### 5. ECONOMIES WITH EFFICIENT EQUILIBRIA

We mentioned earlier that it is possible to give conditions on the economy sufficient to ensure that at least one equilibrium is efficient. We now turn to these conditions. One of them requires little comment. It is perfectly clear from Figure 2 that the problem with which we are concerned can arise only if community indifference curves intersect. There is of course a substantial literature (see for example Gorman (1953) and Muellbauer (1976)) on the properties of these curves, and the authors there present conditions sufficient to

ensure that they do not intersect. It is a simple matter, which we leave to the reader, to verify that such conditions are also sufficient to ensure the existence of at least one efficient equilibrium in an economy of the type discussed here. (Details are given in Brown and Heal (1978).)

There is an alternative approach to this issue, which consists of identifying the economies which are in a certain sense topologically equivalent to convex economies.

To develop this idea further some extra notation is needed. Let  $\mathcal{U}$  be the set of feasible utility vectors  $U$  for the economy. We shall denote by  $S(U)$  the set of points in commodity space capable of yielding at least this utility vector—i.e. the set of points on or above the corresponding social indifference curve.

$$S(U) = \{x \in R^n / \exists x_i, U_i(x_i) \geq U_i V_i, \sum_i x_i = x\}$$

where  $U_i$  is the  $i$ th component of  $U$ . It follows from the quasi-concavity of individual utility functions that the  $S(U)$  are convex.

The idea now is to posit the existence of a diffeomorphism  $F$  of  $R^n$  to  $R^n$  which sends both  $Y$  and  $S(U)$  for all feasible  $U$  into convex sets. The image of the original economy under this diffeomorphism is thus a well-behaved convex economy for which all equilibria are efficient. This is the sense in which it is topologically equivalent to a convex economy, and what has to be shown is that the properties of interest are preserved by this topological equivalence.

A final item of notation: we shall denote by  $\partial Z$  the boundary of a set  $Z$ .

**Proposition.** *Let  $\partial Y, \partial S(U), \forall U \in \mathcal{U}$  be  $C^1$  manifolds. Let there exist a  $C^1$  diffeomorphism  $F: R^n \rightarrow R^n$  such that  $F(Y), F(S(U)), \forall U \in \mathcal{U}$  are convex sets. Then every equilibrium is efficient.*

The proof is omitted. This result is perhaps not surprising: efficiency can be viewed as a property of the structure of critical points on a manifold, so that one would expect it to be a topological invariant.

The result enables us to isolate a certain class of non-convex economics as being “nice” and “like” the economies with which we are familiar. However, it is not at all clear how big this class is nor is it clear what conditions would have to be satisfied by individual preferences and production sets for the assumptions of the proposition to hold.

## 6. CONCLUSIONS

We have, as promised, produced a simple and non-paradoxical economy to illustrate a phenomenon first noted by Guesnerie, namely that with increasing returns it may be the case that none of the equilibria satisfying the first order conditions is in fact efficient. It has also been shown that a unilateral transfer of endowments may result in there being at least one efficient equilibrium, and may in consequence make all parties better off.

These facts have, as was observed in the introduction, a number of implications. One is that some endowment patterns are efficient and others inefficient. Hence one cannot regard judgements on equity and efficiency as separable: one could not for example regard a reallocation of endowments as beneficial because it reduces inequality, on the assumption, generally implicit, that efficiency is not significantly affected. The efficiency of the system may be affected substantially. Of course, the possibility of a trade-off between equity and efficiency is widely recognized, but is usually attributed to the disincentive effects of taxation. Our examples suggest that such trade-off may exist even in a first-best world.

As has already been remarked, these facts have implications both for the theory of welfare economics and for its applications in the field of cost-benefit analysis. Of course, they also have implications for pricing policies in regulated increasing-returns industries. A traditional argument is that, provided the losses can be covered in a non-distortionary

fashion, these should price at marginal cost, ensuring, in a first-best world, an efficient allocation of resources. It should now be clear that this argument is incorrect, and not merely because it neglects second-order conditions. Endowments may be such that there is no way of achieving global Pareto optimality by marginal-cost pricing.

Not all non-convex economies are afflicted by these problems, and it is naturally of importance to be able to describe the boundary between those which are and those which are not. In some measure it has been possible to do this, by isolating cases which are well-behaved. However, it is clear that a great deal more work is needed in this field.

There is just one final point to note. Although in a non-convex economy it is not the case that an efficient outcome can be reached from any set of endowments, it is nevertheless still true that, given a point on the utility possibility frontier, there exists some pattern of endowments with an equilibrium yielding this utility vector. This is an immediate corollary of Theorem 3 of Brown and Heal (1979).

*First version received July 1978; final version accepted January 1979 (Eds.).*

This Research was supported in parts by grants from the National Science Foundation and the Ford Foundation to the Universities of Yale and Stanford. We would like to acknowledge the considerable benefits of discussions with Graciela Chichilnisky, Robert Eastwood, Roger Guesnerie, Peter Hammond, Al Klevorick, Rolf Mantell, Kotaro Suzumura and Sidney Winter.

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