The relationship between price and extraction cost for a resource with a backstop technology

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This paper analyzes the optimal depletion policy for a country with a resource which is inexhaustible but available in various grades and at various costs. Cost is assumed to increase with cumulative extraction up to a point, but then to remain constant as a "backstop" supply is reached. This models accurately the supply conditions of minerals which may eventually be extracted from marine sources or crustal rocks. Emphasis is placed on the behavior of prices along an optimal (competitive) path, and it is shown that the price-cost relationship is very different from the standard one of an exponentially growing royalty.

1. Introduction

■ Much has been written recently about the optimal depletion policies for exhaustible resources, and the reasons for this are, of course, fairly obvious. It is perhaps less obvious why one should ask similar questions about resources that are effectively inexhaustible. In fact, the motivation is closely related, for there is evidence that many of the resources commonly regarded as exhaustible are in fact available in effectively unlimited quantities, but under a range of different supply conditions. It appears to be the case that aluminum, manganese, and a number of other metals come into this category: their ores can be extracted from the deposits currently in use, and the metals themselves can be extracted from the sea or from widely available rock formations. Current deposits are much cheaper than the latter two sources, and are also exhaustible; estimates of the amounts available from the latter sources are subject to great error, but suggest that, if society is prepared to pay the price, supplies will be available into the indefinite future (see Nordhaus, 1974, and "Resources and Man"). In this sense there is available for these resources a backstop technology—to use a phrase given prominence by Nordhaus (1973). In his analysis, the extraction of oil from shale served as a backstop against the exhaustion of conventional oil deposits. My aim in this paper is, therefore, to analyze the optimal strategy for consuming a resource whose total availability is infinite,

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but whose cost of extraction is an increasing function of the total already extracted: this cost function is assumed to be bounded above. This seems to describe with tolerable realism situations such as those referred to above, where there are a number of different sources of supply, the cheaper ones being exhaustible, but the most expensive one (typically extraction from seawater or crustal rock formations, or possibly large-scale recycling of past consumption) providing unlimited amounts of the input in question.

During the analysis, particular attention will be paid to the behavior of the prices of the extracted and unextracted resource. We shall analyze the relationship between the rate of change of the price of the extracted resource and the discount rate, and the determinants of the difference between extracted and unextracted prices. In earlier studies (such as Kay and Mirrlees, 1975, and Dasgupta and Heal, 1974) the relationship found here has been very simple:

$$P_x = MEC + r,$$

where P_x is the price of the extracted resource, MEC is its marginal extraction cost, and r is a royalty or rental element which can also be interpreted as the price of the unextracted resource. It is typically argued that r grows exponentially, and in consequence P_x is near MEC initially, but comes to depart substantially and rapidly in the long run. For example, if marginal extraction cost is constant at M, we have

$$P_x = M + r_0e^{\delta t} \text{ or } \frac{P_x - M}{M} = \frac{r_0}{M} \cdot e^{\delta t},$$

where δ is the social rate of time preference. The fact that $P_x - M$ grows exponentially at the discount rate ensures that the present value of the net marginal product of the resource is constant, an obvious necessary condition for efficient use of a fixed stock, and for intertemporal market equilibrium. Conclusions of considerable consequence have been drawn—or at least tentatively inferred—from this price-cost relationship. For example, the fact that for many resources P_x is greatly in excess of MEC when stocks are clearly very large has been used as a basis for arguing that the present price is substantially above the optimal or competitive price and that the resources in question are being overconserved. Kay and Mirrlees (1975, p. 162) have suggested this conclusion, and this line of argument is also fundamental to, though perhaps not quite so explicit in, Nordhaus' conclusion (1973) that the then current price of oil was greatly in excess of its competitive price.

In the present model we shall see that the difference between P_x and MEC is not equal to a royalty (because the resource is, after all, available in unlimited quantities), but arises because of a social cost of extraction not reflected in the MEC: this social cost reflects the fact that present extraction, by raising the total extracted, pushes up future extraction costs. It will be shown that, under the assumption that unit extraction costs as a function of cumulative extraction are bounded above, this social cost falls to zero and consequently the difference between P_x and MEC falls monotonically. This is, of course, the reverse of the situation described earlier, where P_x was near MEC for a long time, departing significantly only for large t. It is easy to give an intuitive explanation for the fact that r declines and so

P_x approaches MEC: if extraction costs are bounded above, the influence of changes in cumulative extraction on such costs declines to zero, and in the present context this removes the only reason for P_x to exceed MEC.

It follows that if the present model captures important aspects of reality by assuming extraction costs to depend on cumulative past extraction, with some upper bound on costs at which an effectively infinite supply becomes available, then a large discrepancy between price and extraction costs might be seen as comforting evidence that the market is working well, rather than as worrying evidence of monopolistic overconservation.

2. The model

■ Except for the conditions of supply of the resource, the model is much as in Dasgupta and Heal (1974): output Y is produced from capital K and resource flows R according to a production function $F(K,R)$ which shows nonincreasing returns to scale and diminishing returns to proportions. The total amount of the resource used by time t is denoted by $z(t)$

$$z(t) = \int_0^t R_\tau d\tau \tag{1}$$

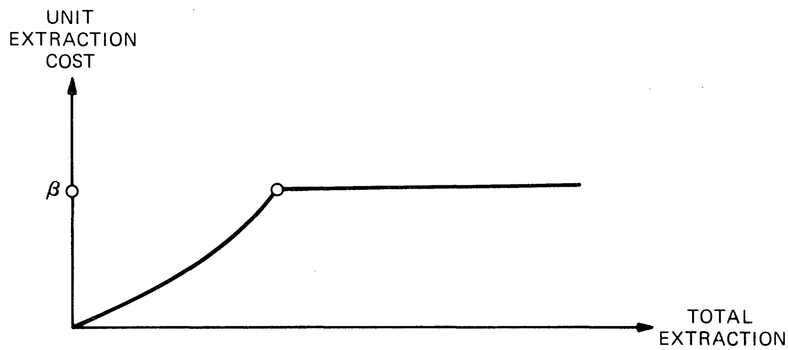
and the cost of extracting a unit of the resource at time t is given by

$$g(z), \text{ with } \frac{\partial g}{\partial z} \equiv g'(z) > 0, \text{ for } 0 \leqq z \leqq \bar{z} \tag{2}$$

$$\beta > 0 \text{ for } z \geqq \bar{z} \text{ where } g(\bar{z}) = \beta.$$

This unit cost function reflects the fact that for $z < \bar{z}$, unit costs rise with cumulative extraction. $\bar{z}$ represents the total of low-cost deposits available, and once these are exhausted it is necessary to switch to a source of supply with high but constant costs. To establish sufficient conditions for optimality, it is convenient to assume that $g(z)$ is a convex function—though necessary conditions can of course be established in any case. Figure 1 shows the sort of cost function we have in mind.

FIGURE 1
UNIT COST FUNCTION



If we let $G(z)$ be the cost function defined by (2), the overall planning problem can now be posed as follows:

$$\text{Max} \int_0^\infty u(C)e^{-\delta t}dt$$

subject to

$$C + \dot{K} = F(K,R) - G(z)R, \tag{3}$$

$$\dot{z} = R,\ z(0) \text{ given, and } C \geqq 0, R \geqq 0,$$

where z is defined by (1). Here $\delta > 0$ is a discount rate, $u(C)$ is a strictly concave function measuring the satisfaction derived from a consumption level C, and the right-hand side of (3) is just output net of extraction costs. The objective is thus to maximize the present value of the benefits derived from consumption over the indefinite future, subject to constraints imposed by the initial conditions and the production and extraction technologies. These latter determine the tradeoff available at any point of time between consumption and capital accumulation. To avoid worrying about corner solutions for C and R, we shall assume:

$$u'(0) = +\infty$$

$$\frac{\partial F(K,0)}{\partial R} = +\infty.$$

It is convenient to solve this problem by solving the following two problems and piecing together the resulting solutions:

$$\text{Max} \int_0^\infty u(C)e^{-\delta t}dt$$

subject to

$$C + \dot{K} = F(K,R) - g(z)R \tag{4}$$

$$\text{Max} \int_0^\infty u(C)e^{-\delta t}dt$$

subject to

$$C + \dot{K} = F(K,R) - \beta R. \tag{5}$$

These correspond to the two regimes of the cost structure defined in (2), and it seems likely that any optimal path must pass through both. (The alternative is that cumulative extraction over the entire horizon does not exceed $\bar{z}$: conditions sufficient to rule this out are self-evident.) Consequently we consider first the solution to problem (4). Applying the maximum principle leads to an examination of the Hamiltonian

$$\mathfrak{H} = u(C)e^{-\delta t} + pe^{-\delta t}\{F(K,R) - g(z)R - C\} + qe^{-\delta t}R,$$

where p and q are the adjoint variables corresponding respectively to the two state variables K and z. Maximizing with respect to the control variables C and R gives as necessary conditions

$$u'(C) = p \tag{6}$$

$$p(F_R - g(z)) = -q. \tag{7}$$

The movement of the adjoint variables is described by

$$\dot{p} - \delta p = -F_K p \tag{8}$$

$$\dot{q} - \delta q = pRg' = pg'\dot{z} = p\frac{dg}{dt}. \tag{9}$$

(6) and (8) are of course familiar conditions, so that only (7) and (9) merit comment. Consider (7) briefly: pF_R is the competitive price of the extracted resource, and $pg(z)$ is the marginal (and average) extraction cost. $-q$ can thus be interpreted as the imputed price of a unit resource concession or of a unit of ore. Differentiating (7) and using (9) and (7),

$$\frac{d}{dt}(PF_R) = \delta pF_R - \delta g(z)p + \dot{p}g(z),$$

or, if $pF_R = \tilde{p}$, $pg(z) = \tilde{c}$, then

$$\frac{\dot{\tilde{p}}}{\tilde{p}} = \delta(1 - \frac{\tilde{c}}{\tilde{p}}) + \frac{\dot{p}}{p}\frac{\tilde{c}}{\tilde{p}}. \tag{10}$$

In simpler models, one obtains an equation relating the rate of change of the competitive market price of the resource to the discount rate: (10) is the generalization of that to the present case. The rate of change of the resource price equals the discount rate times the fraction of the price that is pure rent, plus the rate of change of the output price times the share of costs in price. The rate of capital gain on the extracted resource is thus a weighted average of the discount rate and the rate of change of output prices, the weights being the proportions that royalties and costs contribute to price.

Another insight into the problem may be gained by writing (7) as

$$pF_R = pg(z) - q.$$

Now as z is a nuisance good, q is a negative number. This then implies that the market price of the resource should equal its marginal extraction cost plus a surcharge to reflect the impact that current extraction has on future costs. One might think of $pg(z)$ as the immediate marginal cost, and $pg(z) - q$ as the total marginal cost. Solow and Wan (1975) have referred to $-q$ as a "degradation charge."

If the production function shows constant returns to scale, there is another way in which the necessary conditions may be related to those found elsewhere in the literature: after some manipulation, they can be cast in the form

$$\dot{k} = \sigma f(k) + \frac{f'(k)}{kf''(k)} g(z), \tag{11}$$

where σ is the elasticity of substitution between capital and resources. This is an obvious generalization of equation (1.18) of Dasgupta and Heal (1974).

We now have as much information as is needed about the solution to problem (4), and turn to (5). This is a one-state variable problem whose necessary conditions are

$$u'(C) = p \tag{12}$$

$$F_R(K,R) = \beta \tag{13}$$

$$\dot{p} = p(\delta - F_K(K,R)), \tag{14}$$

where p is the adjoint variable associated with the capital accumulation constraint. (12) and (14) can be combined into

$$\frac{\dot{c}}{c} = \frac{\delta - F_K(K,R)}{\eta},\ \eta = \frac{u''(C)C}{u'(C)} \tag{15}$$

It is easy to see how (13) and (15) determine the evolution of the system. K_0 is of course given, so (13) fixes R_0. This gives $y_0 = \dot{K}_0 + C_0$, and C_0 is chosen to have the maximum value compatible with $C_t > 0$ forever. The existence of such a maximum is assured by standard arguments (see von Weizäcker, 1965 and Heal, 1973) and again by standard procedures one can show that the resulting trajectory satisfies conditions both necessary and sufficient for optimality. For a Cobb-Douglas production function of the form

$$f(k) = \frac{F(K,R)}{R} = k^{\alpha},$$

and an isoelastic utility function, (15) yields

$$C_t = C_0 \exp\left\{\frac{\delta - \alpha\left[\dfrac{\beta}{1-\alpha}\right]^{\frac{\alpha-1}{\alpha}}}{\eta}\right\}t.$$

Consumption thus rises or falls exponentially over time, depending on the relative magnitudes of the parameters: for one set of values, it will remain constant.

All the useful information about the solutions to problems (4) and (5) has now been assembled, and it only remains to link the two together. The following proposition shows how to do this.

Proposition: If on an optimal path there exists $T < \infty$ such that $z_T = \bar{z}$, then

$$F_R(K_T, R_T) = g(z_T)$$

on that path.

Proof: Suppose that on an optimal path $z^* = \bar{z}$ at T, and

$$F_R - g(z^*) \geqq \mu > 0 \;\forall t \in [0, T],$$

where an asterisk denotes the value of a variable on the optimal path. Then we construct an alternative path from the optimal path by raising $R^*(t)$ by $\Delta R > 0$ from some $T' < T$ up to T. The benefits from this change are

$$\int_{T'}^{T} u'(C_t^*)F_R(K_t^*, R_t^*)\Delta R e^{-\delta t} dt$$

and to a first-order approximation the costs are bounded above (because of the discontinuity in g' at z) by

$$\int_{T'}^{T} u'(C_t^*)e^{-\delta t}\{g(z_t^*) + g'(z_t^*)\Delta z_t\}\Delta R dt$$

$$= \int_{T'}^{T} u'(C_t^*)e^{-\delta t}\{g(z_t^*)\Delta R + g'(z_t^*)(t - T')\Delta R^2\}dt.$$

The net gain is

$$\int_{T'}^{T} u'e^{-\delta t}\Delta R\{F_R - g(z^*) - g'(z^*)(t - T')\Delta R\}dt$$

$$\geqq \int_{T'}^{T} u'e^{-\delta t}\Delta R\{\mu - g'(z^*)(t - T)'\Delta R\}dt.$$

Now let $T - T' = \Delta t$, a small number:

$$\text{net gain} \geqq u'e^{-\delta t}\Delta R\{\mu - g'(z^*)\Delta t\Delta R\}\Delta t = u'e^{-\delta t}\{\Delta R\mu\Delta t - g'(z^*)(\Delta t)^2(\Delta R)^2\}$$

and this can be made strictly positive by choosing Δt sufficiently small. The proposition is thus established by contradiction. *Q.E.D.*

On consideration, the proposition is not at all surprising. It is clear that as the economy comes nearer to exhausting the low-cost resource stocks for which extraction cost depends on output, the social costs of extraction—represented by $-q$—decline, because of the shrinkage of the residual which they will affect. The proposition asserts that they will reach zero just as the low-cost stocks are exhausted.

It is now straightforward to describe the complete optimal path. There is an initial period during which the lower-cost stocks are exhausted: the behavior of the economy is described by (6) to (9) during this period, with prices related to costs according to (10). The initial conditions are so chosen that the difference between prices and extraction costs, given by $-q$, declines according to (9) and just reaches zero as the lower-cost stocks are exhausted. At this point royalties become zero. From then on extraction costs of the resource always equal its price, and the economy behaves according to (12) to (14).

3. Conclusion

■ We have shown that the relationship between the price of a resource and its marginal extraction cost along an optimal path (or, equivalently, along a path realized by an intertemporal competitive equilibrium) depends on the nature of the extraction technology. If one assumes that extraction costs are intertemporally linked via the influence of cumulative extraction, and in addition, that they are bounded above because of the existence of a backstop technology, then this price-extraction cost relationship differs strikingly from that usually postulated. Instead of the price's starting near marginal extraction cost, and drawing exponentially away as royalty elements come to dominate, we find exactly the reverse. The price starts a long way above *MEC,* but falls towards it. Of course, the differences are easily explained in terms of the differences in the assumptions underlying the models. But what is important is the implication of these distinctions for the way one interprets the striking gaps between resource prices and their extraction costs on which several writers have commented. According to one model, these are clear evidence of overconservation and market failure: according to the other, no such conclusion can be drawn. Naturally, a choice between the models has to be made on the basis of their empirical realism: evidence has already been cited to suggest that the present formulation merits careful consideration. Of course, it is not even necessary to accept the entire model of extraction costs used above to overthrow that part of the conventional wisdom that implies that price will be "near" *MEC*, when stocks are large. For this one needs only to accept that current costs are an increasing function of cumulative extraction: whether this function is bounded or not, it follows immediately that the long-run costs of present extraction drive a further, and possibly very large, wedge between price and current costs.

Before concluding this analysis entirely, it is worth dwelling briefly on the implication of equation (7) that

$$pF_R > pg(z), \; z < \bar{z}.$$

For at least a part of an optimal path, the current benefits of resource use exceed the current costs. Could one reasonably expect a market system to realize such a condition? This depends on the nature of the property rights in the resource. If it is extracted from a concession owned by someone who expects to continue in ownership *ad infinitum*, or at least until the entire low-cost stock is depleted, then it is possible that the condition will be met. Whether or not (7) and (8) are exactly satisfied will depend on the owner's rate of time preference. If, on the other hand, there is no exclusive property right in the resource—as may be the case with marine resources—or if there is a chance of the property right's being terminated before complete extraction of the low-cost source on an optimal path—then profit maximizing extraction will lead to a less than optimal difference between marginal current benefits and costs. For example, if there is no exclusive property right in the resource, and there is free entry and exit into its extraction, firms will expand their activities until $F_R = g(z)$. At this point the average and marginal costs of production will be equated to the resource price, and in consequence the conditions necessary for intertemporal optimality will be violated. For any given level of cumulative extraction z, price will be lower, and extraction greater, than optimal. The extraction program will be forward-biased, with the high-cost supplies reached earlier than is optimal, and the price's beginning lower but rising more rapidly than on an optimal path.

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