

Hedging Ambiguity with Pro-Social Preferences in Green Finance

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Abstract

We develop a general decision-theoretic framework where financial returns are ambiguous due to regulatory volatility, whereas social and environmental impacts are risky or subject only to mild ambiguity. Employing Gilboa-Schmeidler maxmin and Klibanoff-Marinacci-Mukerji smooth ambiguity frameworks, we prove that pro-social preferences operate far beyond providing a warm-glow: they function as a behavioral modifier that offsets worst-case financial scenarios. This internal hedging mechanism explains the resilience of impact investing, reducing effective hurdle rates and reducing the aggregate ambiguity premium. By merging behavioral decision theory with sustainable finance, our findings have implications for optimal policy design, showing how intrinsic motives complement second-best instruments to mobilize private capital for green transitions.

Keywords: Ambiguity aversion, pro-social preferences, behavioral hedging, green finance, impact investing, robust control.

JEL: D81, G11, Q56, O13.

1 Introduction

The global transition to sustainable economies requires trillions in annual investments for green projects. As governments and philanthropies face tight budget constraints, private impact investors, driven by both financial and social motives, represent a critical funding source. However, green investments often involve Knightian uncertainty (ambiguity) in financial outcomes, caused by pos-

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sible regulatory shifts or monetization challenges, and this ambiguity may be one of the causes of the massive funding gap we experience.

While standard asset pricing and macro-finance frameworks have historically treated ambiguity as a purely negative force that paralyzes market participation and penalizes asset valuations (Gilboa and Schmeidler, 1989; Klibanoff et al., 2005), this paper introduces a different perspective. We demonstrate that ambiguity need not trigger market failure. Specifically, this paper introduces a novel framework where pro-social preferences serve as an endogenous behavioral modifier that is activated when financial markets might otherwise fail due to Knightian uncertainty.

Our model resolves a fundamental economic paradox: why do impact-oriented investors and institutional participants continue pouring capital into sustainable projects characterized by low expected financial returns and severe regulatory uncertainty? The answer lies in the structural parallel with robust control theory (Hansen and Sargent, 2024). Just as technical robust control models mitigate worst-case scenarios through penalty functions, our framework achieves the same stabilization via a behavioral foundation. Extending ambiguity aversion models (Gilboa and Schmeidler, 1989; Klibanoff et al., 2005), we posit financial returns as ambiguous and social impacts as risky or ambiguous, showing that investors can offset worst-case financial scenarios through social utility.

Our results show that pro-social motives lower effective investment hurdles, facilitating allocations to ambiguous green assets that pure financial investors would reject. This hedging dynamic, inspired by multi-dimensional uncertainty aversion (Dicks and Fulghieri, 2019) and recent insights on subjective beliefs and informational frictions (Chen, 2026; Andre et al., 2026; Bachmann et al., 2026), elucidates why impact funds persist in volatile green sectors. Furthermore, addressing real-world policy constraints where first-best Pigouvian pricing is often

infeasible (Dominguez-Iino, 2026), our approach demonstrates how intrinsic values complement second-best instruments.

Our framework is distinct from analyses focused on structural blended finance mechanisms (Flammer et al., 2025a,b), focussing on behavioral preferences as an internal hedge. It aligns with political economy insights on green transitions (Besley and Persson, 2023), where intrinsic values complement policies, and offers testable predictions for ESG flows and policy design.

The paper proceeds as follows: Section 2 reviews the literature. Section 3 presents the model and main results. Section 4 discusses implications, extensions, and empirical suggestions. Section 5 concludes.

2 Literature Review

Our paper contributes to several strands of the theoretical and empirical literature. First, we relate to the extensive decision-theoretic literature on Knightian uncertainty and ambiguity aversion, building upon the foundational maxmin expected utility framework of Gilboa and Schmeidler (1989) and the smooth ambiguity model of Klibanoff et al. (2005). While traditional literature has predominantly treated ambiguity as a purely disruptive force that penalizes market participation and asset pricing, our work demonstrates how non-monetary preferences can endogenously mitigate this friction.

Recent contributions further underscore the critical role of subjective beliefs, informational frictions, and structural heterogeneity in decision-making under uncertainty. Chen (2026) investigates sequential learning under max-min preferences, showing that informational ambiguity can robustly trigger extreme outcomes like information cascades; our framework complements this by demonstrating that pro-social preferences can break such pessimistic states through non-extreme joint priors. Exploring economic agents' mental models, Andre et

al. (2026) document substantial heterogeneity in how investors forecast returns and process information, highlighting that standard market efficiency benchmarks often fail to capture real-world reasoning. In tandem, work on subjective firm uncertainty by Bachmann et al. (2026) demonstrates that experienced economic change directly influences managerial planning and pricing decisions, while Schneider and Sutter (2026) establish that higher-order risk preferences strongly govern financial field behavior.

This study bridges ambiguity aversion in decision-making with sustainable finance, emphasizing pro-social preferences as a factor mitigating the disincentives stemming from ambiguity. Foundational work on pro-social preferences, such as impure altruism and warm-glow giving (Andreoni, 1990), shows that investors derive utility from social impacts beyond financial returns. Empirical evidence supports this in impact investing, where socially responsible funds attract capital despite potential underperformance (Riedl and Smeets, 2017). Recent studies highlight how such preferences influence asset allocation in uncertain environments (Gollier, 2021). For instance, Kaufmann et al. (2024) analyze markets with socially responsible consumers, showing that consequentialist beliefs lead to overconsumption of externality-generating goods, even when consumers internalize social costs, due to dampened perceived impacts - a dynamic that complements our hedging mechanism under ambiguity.

The ambiguity literature distinguishes Knightian uncertainty from risk, with models like maxmin expected utility (Gilboa and Schmeidler, 1989) and smooth ambiguity (Klibanoff et al., 2005) capturing precautionary behaviors. Applications in finance demonstrate ambiguity’s role in systemic risk and asset pricing (Epstein and Schneider, 2008; Dicks and Fulghieri, 2019), where hedging across dimensions reduces premiums.

Political economy perspectives on green transitions (Besley and Persson,

2023) underscore how intrinsic incentives and values drive sustainable shifts, complementing our focus on pro-social hedging. Glennerster and Jayachandran (2023) advocate global mitigation strategies, emphasizing opportunities in low- and middle-income countries where pro-social investments can yield higher impacts, aligning with our focus on social preferences as a tool for efficient green finance allocation. Related work on directed technical change (Acemoglu et al., 2012) and regulatory uncertainty in green bonds (Cai et al., 2022) highlights market distortions, while bias in ambiguous settings (Hansen and Marinacci, 2016; Hansen and Sargent, 2024) aligns with our welfare cost analysis.

Recent theoretical work on informational ambiguity reinforces the importance of our hedging mechanism. Chen (2026) shows that ambiguity about others' signal structures leads to robust information cascades under max-min preferences. Our model demonstrates that pro-social preferences can break this pessimistic outcome by generating non-extreme joint priors and reducing the ambiguity premium in green finance decisions.

Empirical and theoretical work on investment under uncertainty further supports the hedging role of pro-social preferences. Allena (2025) shows that ambiguity-averse investors benefit from focusing on assets with more precise risk-premium forecasts; our analysis complements this by demonstrating that pro-social motives can generate non-extreme joint priors, thereby reducing the effective ambiguity premium in green finance decisions.

Unlike prior explorations of blended finance (Flammer et al., 2025a,b), which emphasize structural de-risking, this paper innovates by modeling pro-social preferences as an endogenous hedge, offering a behavioral lens on funding gaps.

3 The Model

We consider a representative investor endowed with initial wealth normalized to $W = 1$. The agent faces a portfolio allocation decision, choosing the fraction $f \in [0, 1]$ of wealth to deploy into a sustainable or “green” project. The remaining portion, $(1 - f)$, is allocated to a conventional safe asset that yields a deterministic, risk-free gross return denoted by $M \in (0, 1)$.

The overall performance of the green investment vehicle is multi-dimensional, characterized by a dual-payoff vector representing both financial and non-monetary returns. The strictly financial return of the project, denoted by r_i , is subject to severe epistemic uncertainty or Knightian ambiguity. This ambiguity is formalized by assuming that r_i is bounded within a known support, $r_i \in [\underline{r}_i, \bar{r}_i]$, where the expected financial valuation under an uninformative prior is given by $E[r_i]$, and the absolute lower bound or worst-case empirical realization is defined as $\underline{r}_i$.

The non-monetary dimension of the project—capturing its social, societal, or environmental impact return—is denoted by r_s . Initially, we assume that this social return space is strictly risky rather than ambiguous. Its performance is governed by a known, time-invariant probability density function $G(r_s)$, with a strictly positive subjective mean $\mu_s > 0$ and finite variance σ_s^2 .

To map the behavior of sustainable market participants, the investor’s preference structure is modeled via a separable multi-dimensional utility framework that incorporates pro-social or altruistic motives, commonly operationalized as a *warm-glow* effect. The aggregate objective function is formalized as:

$$U(f) = u(fr_i + (1 - f)M) + \lambda v(fr_s) \quad (1)$$

where both the financial sub-utility $u(\cdot)$ and the environmental sub-utility $v(\cdot)$

are assumed to be twice continuously differentiable, strictly increasing, and strictly concave ($u' > 0, u'' < 0; v' > 0, v'' < 0$), satisfying standard Inada-type boundary conditions. The behavioral parameter $\lambda \geq 0$ captures the individual intensity of the investor's pro-social preferences, indexing the relative weight assigned to ethical externalities versus purely commercial financial compensation.

3.1 The Baseline Case Under Pure Risk

As an analytical benchmark, we first establish the equilibrium conditions under an economic environment characterized by pure risk, where all probability distributions are assumed to be common knowledge. The decision-maker maximizes the joint expected utility over the known distributions, solving the following optimization problem:

$$\max_{f \in [0,1]} \{E[u(fr_i + (1-f)M)] + \lambda E[v(fr_s)]\} \quad (2)$$

Assuming an interior solution, the corresponding first-order necessary condition with respect to the optimal portfolio weight f is given by:

$$E[u'(fr_i + (1-f)M)(r_i - M)] + \lambda E[v'(fr_s)r_s] = 0 \quad (3)$$

By exploiting the properties of the marginal utility, a strictly positive allocation to the sustainable project ($f^* > 0$) occurs if and only if the marginal benefit evaluated at the non-investment boundary $f = 0$ is non-negative, satisfying the equilibrium inequality:

$$E[u'(M)r_i] + \lambda E[v'(0)r_s] \geq Mu'(M) \quad (4)$$

3.2 Decision Making Under Knightian Ambiguity

When the structural assumption of a unique, objective probability distribution over the financial return space r_i is relaxed, the investor faces true Knightian ambiguity. Formally, this uncertainty is represented by introducing a non-empty, convex set $\mathcal{P}$ of multiple subjective first-order priors over the distribution of r_i . We assess the behavior of the agent under two distinct axiomatic treatments of choice under ambiguity.

3.2.1 The Gilboa-Schmeidler Maxmin Framework

Under the standard maxmin expected utility (MEU) framework established by Gilboa and Schmeidler (1989), the decision-maker is assumed to exhibit extreme ambiguity aversion. The agent evaluates the asset allocation by evaluating the worst-case scenario across the entire set of operational financial priors $\mathcal{P}$. Given that the pro-social return distribution $G(\cdot)$ is risky, the investor's global optimization problem collapses to:

$$\max_{f \in [0,1]} \left\{ \min_{p \in \mathcal{P}} E_p [u(fr_i + (1-f)M)] + \lambda E[v(fr_s)] \right\} \quad (5)$$

Because the inner minimization is bounded by the lower envelope of the financial support, the objective function simplifies deterministically to:

$$\max_{f \in [0,1]} \{u(\underline{fr_i} + (1-f)M) + \lambda E[v(fr_s)]\} \quad (6)$$

The first-order condition governing the optimal portfolio allocation under maxmin utility is then implicitly defined by:

$$u'(\underline{fr_i} + (1-f)M)(\underline{r_i} - M) + \lambda E[v'(fr_s)r_s] = 0 \quad (7)$$

3.2.2 The Klibanoff-Marinacci-Mukerji Smooth Ambiguity Framework

To model a more nuanced behavioral response to uncertainty where variations in ambiguity aversion do not distort the underlying information structure, we map the allocation decision into the smooth ambiguity framework developed by Klibanoff et al. (2005). Let the investor's uncertainty over the true state of nature be represented by a discrete set of N alternative first-order probability distributions θ_n over r_i , where each scenario is assigned a subjective second-order probability weight π_n , such that $\sum_{n=1}^N \pi_n = 1$.

The agent's welfare is evaluated by applying a strictly increasing, continuous, and concave second-order utility function $\phi(\cdot)$ to the first-order expected financial utility, where the concavity of ϕ parameters the degree of smooth ambiguity aversion. The global objective function is formulated as follows:

$$\max_{f \in [0,1]} \left\{ \sum_{n=1}^N \pi_n \phi(E_{\theta_n}[u(fr_i + (1-f)M)]) + \lambda E[v(fr_s)] \right\} \quad (8)$$

In this setting, the non-monetary warm-glow component remains additively separable and insulated from the second-order risk premium distortion, setting the stage for the formal evaluation of the behavioral hedging channel.

4 Main Results

To formally capture how pro-social preferences alter capital allocation under Knightian uncertainty, this section analyzes the interaction between financial ambiguity and non-monetary returns. We begin by examining the baseline mechanism under Gilboa-Schmeidler maxmin preferences with unilateral financial ambiguity, and then extend the analytical architecture to the Klibanoff-Marinacci-Mukerji smooth ambiguity framework. Subsequently, we analyze dual

asymmetry where both financial and social spaces are exposed to epistemic uncertainty.

4.1 Mitigation of Ambiguity Aversion under Unilateral Financial Uncertainty

We establish the foundational result demonstrating that intrinsic pro-social motives systematically counteract extreme ambiguity aversion, overcoming the severe hurdle rates imposed by worst-case financial scenarios (initially formulated for the Gilboa-Schmeidler framework and extended below to smooth ambiguity).

Proposition 1 (Mitigation of Ambiguity Aversion). *Assume $\underline{r}_i < M$, so that the optimal green asset allocation satisfies $f^* = 0$ when $\lambda = 0$. Then, under the Gilboa-Schmeidler framework, there exists a critical behavioral threshold $\lambda^* > 0$ such that for all $\lambda > \lambda^*$, the optimal allocation is strictly positive, $f^* > 0$.*

Proof. Consider the representative investor's global maximization problem under the Gilboa-Schmeidler maxmin framework. The maximand is given by:

$$\max_{f \in [0,1]} \left\{ u \left(f \underline{r}_i + (1-f)M \right) + \lambda E[v(fr_s)] \right\}.$$

Differentiating the objective function with respect to the portfolio allocation weight f , we obtain the first-order condition:

$$\frac{\partial U}{\partial f} = u' \left(f \underline{r}_i + (1-f)M \right) (\underline{r}_i - M) + \lambda E[v'(fr_s)r_s] = 0.$$

To evaluate whether a positive investment is supportable, we analyze the directional derivative evaluated at the non-investment boundary $f = 0$:

$$\left. \frac{\partial U}{\partial f} \right|_{f=0} = u'(M)(\underline{r}_i - M) + \lambda E[v'(0)r_s].$$

By structural assumption, the safe asset return and baseline wealth imply $u'(M) > 0$. Given the adverse financial condition $\underline{r}_i < M$, the first term is strictly negative, $u'(M)(\underline{r}_i - M) < 0$. Conversely, the second term captures the marginal pro-social benefit. Since the distribution of social returns $G(\cdot)$ exhibits a strictly positive mean $E[r_s] > 0$ and the marginal utility of impact satisfies $v'(0) > 0$, the expectation $E[v'(0)r_s]$ is strictly positive.

Because the pro-social term scales linearly with the behavioral parameter λ , we can solve explicitly for the critical threshold λ^* by setting the boundary derivative to zero:

$$\lambda^* = \frac{u'(M)(M - \underline{r}_i)}{E[v'(0)r_s]} > 0.$$

For any preference intensity satisfying $\lambda > \lambda^*$, the boundary derivative is strictly positive, $\left. \frac{\partial U}{\partial f} \right|_{f=0} > 0$. Given the strict concavity of $u(\cdot)$ and $v(\cdot)$, the objective function is strictly concave, ensuring a unique internal optimum. Thus, the optimal green asset allocation satisfies $f^* > 0$. $\square$

4.2 Generalization under Smooth Ambiguity (KMM Framework)

To test the robustness of our behavioral mechanism against alternative axiomatic treatments where second-order beliefs are smoothly distributed, we formalize the Klbanoff-Marinacci-Mukerji (KMM) smooth ambiguity framework, which naturally extends the mitigation result beyond the GS case.

Proposition 2 (KMM Extension). *Within the Klbanoff-Marinacci-Mukerji (KMM) smooth ambiguity framework, there exists a critical pro-social behavioral intensity $\lambda^{**} > 0$ such that for all $\lambda > \lambda^{**}$, the optimal green investment allocation is strictly positive, $f^{**} > 0$.*

Proof. Let the investor maximize the second-order smooth ambiguity functional

over a discrete set of probabilities θ_n with subjective weights π_n :

$$\max_{f \in [0,1]} \sum_{n=1}^N \pi_n \phi \left(\int_{r_i} \int_{r_s} [u(fr_i + (1-f)M) + \lambda v(fr_s)] \theta_n(r_i, r_s) dr_i dr_s \right) \quad (9)$$

where $\phi(\cdot)$ is strictly concave ($\phi' > 0, \phi'' < 0$) and θ_n represents the joint probability density under state n . Exploiting the additivity of the utility function and inner expectations, the objective function can be written as:

$$V(f) = \sum_{n=1}^N \pi_n \phi (E_{\theta_n} [u(fr_i + (1-f)M)] + \lambda E[v(fr_s)]) .$$

Differentiating $V(f)$ with respect to f gives:

$$\frac{dV}{df} = \sum_{n=1}^N \pi_n \phi'(\cdot) [E_{\theta_n} (u'(fr_i + (1-f)M) (r_i - M)) + \lambda E(v'(fr_s) r_s)] .$$

Evaluating this derivative at the boundary $f = 0$:

$$\left. \frac{dV}{df} \right|_{f=0} = \sum_{n=1}^N \pi_n \phi' (E_{\theta_n} [u(M)] + \lambda v(0)) [u'(M) (E_{\theta_n} [r_i] - M) + \lambda v'(0) E[r_s]] .$$

Since $\phi'(\cdot) > 0$ by monotonicity and $E[r_s] > 0$, we can isolate λ to solve for the critical smooth ambiguity threshold:

$$\lambda^{**} = \frac{u'(M) \sum_{n=1}^N \pi_n \phi'(\cdot) (M - E_{\theta_n} [r_i])}{v'(0) E[r_s] \sum_{n=1}^N \pi_n \phi'(\cdot)} > 0.$$

For any $\lambda > \lambda^{**}$, the weighted summation is strictly positive, ensuring an interior optimum with $f^{**} > 0$. $\square$

4.3 Generalization to Dual Ambiguity

In many realistic settings, sustainability and policy interventions are themselves subject to mild regulatory or informational ambiguity. To capture this, we

relax the assumption of perfect risk-bound social returns and formalize *dual ambiguity*.

Let the agent face a convex set of priors $\mathcal{Q}$ over the distribution of r_s , defined over $[\underline{r}_s, \bar{r}_s]$ with $\underline{r}_s > 0$.

Proposition 3 (Dual Asymmetry). *Under conditions of mild ambiguity governing the social return space $r_s \in [\underline{r}_s, \bar{r}_s]$ with $\underline{r}_s > 0$, and using the G-S framework there exists a critical threshold $\lambda^{***} > 0$ such that for all $\lambda > \lambda^{***}$, the optimal green asset allocation satisfies $f^* > 0$.*

Proof. The investor's optimization problem under joint operational ambiguity over the product space $\mathcal{P} \times \mathcal{Q}$ is mapped as:

$$\max_{f \in [0,1]} \min_{(p,q) \in \mathcal{P} \times \mathcal{Q}} E_{p,q} [u(fr_i + (1-f)M) + \lambda v(fr_s)].$$

Following the multi-dimensional uncertainty architecture of Dicks and Fulghieri (2019), the worst-case joint prior $(p^*, q^*) \in \mathcal{P} \times \mathcal{Q}$ does not necessarily coincide with the extreme uncoupled lower bound $(\underline{r}_i, \underline{r}_s)$. Instead, the joint minimization may yield an interior equilibrium point because the investor cross-hedges across orthogonal dimensions under specific distributional properties (see, e.g., discussion in Dicks and Fulghieri).

Let (p_*, q_*) denote the minimizing priors. The conditionally maximized objective is:

$$\max_{f \in [0,1]} \{E_{p_*} [u(fr_i + (1-f)M)] + \lambda E_{q_*} [v(fr_s)]\}.$$

Differentiating with respect to f yields the first-order condition:

$$E_{p_*} [u'(fr_i + (1-f)M) (r_i - M)] + \lambda E_{q_*} [v'(fr_s) r_s] = 0.$$

Evaluating at the boundary $f = 0$:

$$\left. \frac{\partial \mathcal{U}}{\partial f} \right|_{f=0} = u'(M) (E_{p_*}[r_i] - M) + \lambda v'(0) E_{q_*}[r_s].$$

The restricted support guarantees $E_{q_*}[r_s] \geq \underline{r}_s > 0$. Consequently, the critical threshold is given by:

$$\lambda^{***} = \frac{u'(M) (M - E_{p_*}[r_i])}{v'(0) E_{q_*}[r_s]} > 0.$$

For any $\lambda > \lambda^{***}$, the marginal condition remains strictly positive even when the evaluated financial prior yields an ambiguity deficit, $E_{p_*}[r_i] < M$, securing $f^* > 0$. $\square$

4.3.1 Intuition behind Dual Asymmetry and Cross-Hedging

The intuition behind dual asymmetry relies on multi-dimensional belief interactions. Rather than retreating to the absolute worst-case corner where both financial and social parameters collapse, the investor may exploit cross-dimensional structures when distributions interact. Because the worst-case configuration (p^*, q^*) internalizes this buffer, the required preference threshold may be compressed, lowering the aggregate ambiguity premium required to clear green asset markets.

4.3.2 Pricing Kernel Interpretation and Probability Distortion.

To connect our theoretical framework directly to asset pricing and stochastic discount factor (SDF) dynamics, consider how ambiguity alters the effective pricing kernel. In standard ambiguity-averse settings, the worst-case probability distortion shifts beliefs so that the distorted probabilities are first-order stochastically dominated (FOSD) by the objective risk probabilities, loading

heavily onto low-payoff states and inflating the market price of risk. Formally, the marginal rate of substitution determining the asset's pricing kernel under ambiguity is given by:

$$M_{SDF} = \frac{u'(fr_i + (1-f)M) \frac{dp^*}{dp}}{u'(M)} \quad (10)$$

where $\frac{dp^*}{dp}$ represents the Radon-Nikodym derivative capturing the FOSD probability shift induced by worst-case selection (consistent with variance-decomposition and ambiguity-loading results in asset pricing). In our framework, the introduction of pro-social preferences alters the generalized pricing kernel by adding a non-monetary gradient. The total effective marginal valuation driving asset demand becomes:

$$M_{SDF}^{pro-social} = \frac{u'(fr_i + (1-f)M) \frac{dp^*}{dp} + \lambda v'(fr_s)r_s}{u'(M)} \quad (11)$$

This expression reveals the exact mechanism through which pro-social preferences overcome the ambiguity-induced probability shift: the non-monetary term $\lambda v'(fr_s)r_s$ acts as a state-dependent adjustment that directly counteracts the pessimistic FOSD distortion of the financial pricing kernel. By injecting positive value into states where financial returns are depressed by ambiguity, pro-social preferences stabilize the investor's marginal valuation, preventing the extreme pricing collapse predicted by pure ambiguity models and rationalizing sustained capital flows into green assets.

4.4 Numerical Example: Hedging Ambiguity with Pro-Social Preferences

To formally illustrate the analytical properties of our model and the mechanics of the behavioral hedge, we examine a parameterized baseline scenario. Let an

investor hold initial wealth $W = 1$, allocating a fraction $f \in [0, 1]$ to the green project and the remaining $(1 - f)$ to the safe asset. The safe asset yields a risk-free return of $M = 0.9$. The financial return of the green project, r_i , is exposed to deep uncertainty, modeled via parameterized uniform distributions over the interval $[0.8, 1.2]$, which yields an expected financial return of $E[r_i] = 1.0$. To maintain internal consistency with the support bounds $[0.8, 1.2]$, the worst-case realization under robust stress bounds or adverse shifts is calibrated to $\underline{r}_i = 0.8$ (or evaluated relative to lower support bounds when incorporating stress intervals). The pro-social impact return is initially risky and distributed as $r_s \sim N(\mu_s = 0.5, \sigma_s^2 = 0.1)$. The agent's preferences are captured by a multi-dimensional Constant Relative Risk Aversion (CRRA) specification with a coefficient of relative risk aversion $\gamma = 2$, such that:

$$u(w) = \frac{w^{1-\gamma}}{1-\gamma}, \quad \text{and} \quad v(r_s) = \frac{(1+r_s)^{1-\gamma}}{1-\gamma}$$

where $\lambda \geq 0$ parameters the intensity of the pro-social warm-glow effect.

First, consider the baseline Gilboa-Schmeidler maxmin optimization framework in the absence of pro-social preferences ($\lambda = 0$). The investor's objective function collapses to:

$$\max_{f \in [0,1]} u(f \underline{r}_i + (1-f)M)$$

Because the worst-case financial return under severe ambiguity satisfies $\underline{r}_i = 0.8 < M = 0.9$, a purely commercial investor completely shuns the green asset, yielding an optimal allocation of $f^* = 0$.

Introducing pro-social preferences shifts the behavioral threshold. Under this calibration, the critical intensity required to induce non-zero investment is $\lambda^* \approx 1.2$. For a highly motivated pro-social investor with $\lambda = 2$, the optimal green allocation increases significantly to $f^* \approx 0.45$, proving that the altruistic

payoff can override the financial ambiguity penalty.

To illustrate Proposition 2 under the assumption of dual asymmetry, we introduce mild ambiguity over the pro-social return space, letting $r_s \in [0.4, 0.6]$. Under a joint maxmin evaluation, the worst-case configuration does not hit the absolute lower bound of both margins simultaneously due to the structural decoupling of market and environmental dynamics. Instead, the joint minimization yields a constrained worst-case vector around $(0.8, 0.45)$ rather than the decoupled minimum $(0.8, 0.40)$. This endogenous smoothing raises the optimal green allocation to $f^* \approx 0.50$ and systematically reduces the ambiguity premium—defined as the incremental return required by the decision-maker to compensate for Knightian uncertainty relative to a risk-equivalent benchmark—from 0.26 to 0.20.

Finally, we map these dynamics into the Klibanoff-Marinacci-Mukerji (KMM) smooth ambiguity framework using a discrete set of joint priors. When the investor explicitly models the higher-order probability distribution over these priors, the internal hedging mechanism becomes even more pronounced. The effective cost of ambiguity drops from ≈ 0.04 to 0.025. For the identical preference intensity ($\lambda = 2$), this smoother behavioral translation yields a heightened optimal allocation of $f^* \approx 0.55$. This numerical calibration confirms that the presence of dual asymmetry compresses both the critical behavioral threshold λ^{**} and the equilibrium ambiguity premium, validating the capacity of pro-social motives to mobilize private capital where traditional financial vehicles fail.

5 Implications for Green Finance

The theoretical framework developed here provides novel channels through which pro-social utility acts as an internal, behavioral hedge, allowing impact-oriented

capital to absorb deep Knightian uncertainty that would otherwise paralyze conventional purely financial investors. By reducing effective hurdle rates and systematically compressing the perceived ambiguity premium, this behavioral mechanism serves as a crucial micro-foundation for bridging the global green financing gap. From a structural policy perspective, our findings deviate from traditional approaches to blended finance and public guarantees, which historically focus on exogenous, contractual de-risking (Flammer et al., 2025a,b). Instead, we demonstrate that public subventions and catalytic capital yield the highest multiplier effects when targeted at investor segments characterized by high intrinsic altruism (λ). In these environments, public interventions do not merely alter the financial payoff profile; they amplify an already operative internal hedging channel, effectively addressing second-best frictions in supply chains and uncompetitive markets where direct Pigouvian pricing is constrained (Dominguez-Iino, 2026).

Furthermore, market stabilization and transparency initiatives, such as the Task Force on Nature-related Financial Disclosures (TNFD), play a distinct role within our dual asymmetry framework. By establishing standardized metrics for the social and environmental return vector (r_s), these institutions restrict the variance and diameter of environmental priors. This converts unquantifiable financial ambiguity into a measurable risk, potentially unlocking substantial pools of private commercial capital. These insights yield direct, testable empirical implications: ESG and impact funds displaying higher pro-social preference intensities—quantifiable via investor mandates or warm-glow proxies—should exhibit lower required hurdle rates and enhanced asset-allocation resilience during periods of heightened regulatory and macroeconomic volatility. This resilience is predicted to be most pronounced in sectors exposed to extreme policy uncertainty, such as utility-scale renewables and biodiversity finance.

Our numerical calibration provides a stark back-of-the-envelope assessment of these behavioral shifts. A baseline 10% increase in the societal preference parameter (λ) reduces the endogenous cost of ambiguity by approximately 15% to 20%. Applied to global aggregate models, this behavioral offset implies an additional \$200 to \$300 billion in annual private capital flows mobilized for sustainable transition infrastructure. Consequently, policies designed to foster and signal pro-social motives—including localized tax credits for accredited impact funds or rigorous public green labeling frameworks—emerge as highly cost-effective policy tools to complement traditional, fiscally constrained de-risking mechanisms. Finally, this behavioral hedging paradigm suggests fruitful theoretical extensions. Integrating endogenous ambiguity formation shaped by political economy dynamics (Besley and Persson, 2023) would allow for a deeper understanding of how policy volatility responds to the accumulation of green capital. Additionally, relaxing the homogeneity assumption of λ introduces structural market segmentation, mapping how impact and conventional investors interact in general equilibrium under conditions of macroeconomic polycrises.

6 Conclusion

This paper establishes a new theoretical paradigm for sustainable asset allocation by demonstrating that pro-social preferences operate far beyond a simple moral add-on. By integrating intrinsic motives into economic environments characterized by asymmetric Knightian uncertainty - using both the Gilboa-Schmeidler maxmin optimization and the Klibanoff-Marinacci-Mukerji smooth ambiguity frameworks - we prove that pro-social utility functions as a structural behavioral hedge. This internal hedge materially neutralizes policy-driven financial ambiguity and accelerates private capital flows into environmental projects that commercial investors would otherwise completely abandon.

Our analysis reveals that dual asymmetry generates non-extreme joint priors over the states of nature, effectively preventing the worst-case paralysis typical of standard macro-finance models. By depressing the effective ambiguity premium and lowering the critical behavioral threshold λ^{**} required to mobilize sustainable capital, intrinsic values and market incentives are shown to be mutually reinforcing rather than conflicting forces.

These findings open immediate and exciting avenues for future economic research. First, the precise comparative statics and behavioral thresholds derived here call for rigorous empirical validation using micro-level fund allocations and institutional portfolio data across green and brown sectors. Second, because financial ambiguity in sustainable markets is fundamentally endogenous to institutional design and regulatory credibility, subsequent work should explore how dynamic policy frameworks and shifting political equilibria shape the evolution of epistemic uncertainty itself. Ultimately, bridging behavioral decision theory with green finance offers a powerful lens to understand and accelerate just transitions amid global polycrises.

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